Public Economics, Welfare and Institutions

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Production Externalities

Non-cooperative equilibrium

Taking x_1 as given, the beekeeper chooses x_0 so as to maximize his profit given by

$$\begin{aligned}\Pi_0(x_0, x_1) &= y_0(x_0, x_1) - c_0 x_0 \\ &= 3\alpha_0 x_0^{\frac{1}{3}} x_1^{\frac{1}{3}} - c_0 x_0\end{aligned}$$

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The first-order condition is

$$\frac{\partial \Pi_0(x_0, x_1)}{\partial x_0} = \alpha_0 x_0^{-\frac{2}{3}} x_1^{\frac{1}{3}} - c_0 = 0 \tag{1}$$

This yields

$$R_0(x_1) = \left(\frac{\alpha_0}{c_0}\right)^{\frac{3}{2}} x_1^{\frac{1}{2}}$$

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This yields

$$R_0(x_1) = \left(\frac{\alpha_0}{c_0}\right)^{\frac{3}{2}} x_1^{\frac{1}{2}}$$

Similarly, we derive $R_1(x_0)$

$$R_1(x_0) = \left(\frac{\alpha_1}{c_1}\right)^{\frac{3}{2}} x_0^{\frac{1}{2}}$$

Production Externalities

Non-cooperative equilibrium

At equilibrium, $x_0^* = R_0(x_1^*)$ and $x_1^* = R_1(x_0^*)$. This yields

$$x_0^* = \frac{\alpha_1}{c_1}\left(\frac{\alpha_0}{c_0}\right)^2 \quad \textit{and} \quad x_1^* = \frac{\alpha_0}{c_0}\left(\frac{\alpha_1}{c_1}\right)^2$$

Production Externalities

Non-cooperative equilibrium

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$$x_0^* = \frac{\alpha_1}{c_1}\left(\frac{\alpha_0}{c_0}\right)^2 \quad \textit{and} \quad x_1^* = \frac{\alpha_0}{c_0}\left(\frac{\alpha_1}{c_1}\right)^2$$

These equilibrium choices yield the following honey production

$$y_0^* = 3\frac{\alpha_0^2\alpha_1}{c_0c_1}$$

while the fruit merchant harvests the quantity

$$y_1^* = 3\frac{\alpha_1^2\alpha_0}{c_0c_1}$$

Production Externalities

Non-cooperative equilibrium

The equilibrium profits are given by $\Pi_0^* = \Pi_0(x_0^*, x_1^*)$ and $\Pi_1 = \Pi_1(x_0^*, x_1^*)$. Thus,

Non-cooperative equilibrium profits

$$\Pi_0^* = 2\frac{\alpha_0^2\alpha_1}{c_0c_1} \quad \textit{and} \quad \Pi_1^* = 2\frac{\alpha_1^2\alpha_0}{c_0c_1}$$

Production Externalities

Cooperative equilibrium

The joint profits $\bar{\Pi}(x_0, x_1) = \Pi_0(x_0, x_1) + \Pi_1(x_0, x_1)$ and write

$$\bar{\Pi}(x_0, x_1) = 3(\alpha_0 + \alpha_1)x_0^{\frac{1}{3}} x_1^{\frac{1}{3}} - c_0 x_0 - c_1 x_1$$

Production Externalities

Cooperative equilibrium

The joint profits $\bar{\Pi}(x_0, x_1) = \Pi_0(x_0, x_1) + \Pi_1(x_0, x_1)$ and write

$$\bar{\Pi}(x_0, x_1) = 3(\alpha_0 + \alpha_1)x_0^{\frac{1}{3}} x_1^{\frac{1}{3}} - c_0 x_0 - c_1 x_1$$

The choices $\bar{x}_0$ and $\bar{x}_1$ that maximize $\bar{\Pi}(x_0, x_1)$ must satisfy the following first-order conditions

$$\frac{\partial \bar{\Pi}(\bar{x}_0, \bar{x}_1)}{\partial x_0} = (\alpha_0 + \alpha_1)\bar{x}_0^{-\frac{2}{3}} \bar{x}_1^{\frac{1}{3}} - c_0 = 0 \tag{2}$$

and

$$\frac{\partial \bar{\Pi}(\bar{x}_0, \bar{x}_1)}{\partial x_1} = (\alpha_0 + \alpha_1)\bar{x}_1^{-\frac{2}{3}} \bar{x}_0^{\frac{1}{3}} - c_1 = 0 \tag{3}$$

Production Externalities

Cooperative equilibrium

Substituting the term α_0 in equation (1) by $(\alpha_0 + \alpha_1)$ gives equation (2). The optimal choices are thus

$$\bar{x}_0 = \frac{(\alpha_0 + \alpha_1)^3}{c_0^2 c_1} \quad and \quad \bar{x}_1 = \frac{(\alpha_0 + \alpha_1)^3}{c_1^2 c_0}$$

Production Externalities

Cooperative equilibrium

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The resulting production of honey and fruits $\bar{y}_0$ and $\bar{y}_1$ are respectively given by

$$\bar{y}_0 = \frac{3\alpha_0(\alpha_0 + \alpha_1)}{c_0^2 c_1} \quad \textit{and} \quad \bar{y}_1 = \frac{3\alpha_1(\alpha_0 + \alpha_1)}{c_1^2 c_0}$$

Production Externalities

Cooperative equilibrium

The cooperative equilibrium profits

$$\bar{\Pi} = \frac{(\alpha_0 + \alpha_1)^3}{c_0 c_1}$$

Production Externalities

Cooperative equilibrium

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The difference $\Delta\Pi = \bar{\Pi}(\bar{x}_0, \bar{x}_1) - (\Pi_0^* + \Pi_1^*)$ is given by

$$\Delta\Pi = \frac{(\alpha_0 + \alpha_1)}{c_0 c_1}[(\alpha_0 + \alpha_1)^2 - 2\alpha_0\alpha_1] = \frac{(\alpha_0 + \alpha_1)}{c_0 c_1}(\alpha_0^2 + \alpha_1^2) > 0$$

Production Externalities

Cooperative equilibrium

The cooperative equilibrium profits

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A room for Coase...

This suggests that if the the beekeeper and the fruit merchant were to coordinate their activities, they could jointly achieve higher profits as compared to the case where they had act non-cooperatively. Hence, in the absence of transaction costs, they both would be better off if they were to cooperate.

Pareto Optimality and Public Good

Feasible allocations

An allocation (x, M_1, M_2) is *feasible* if it satisfies the following conditions:

- $x = t_1 + t_2$ and $t_1 = 15 - M_1, \quad t_2 = 15 - M_2$
- $t_1 0$, and $t_2 0$

Pareto Optimality and Public Good

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- $x = t_1 + t_2$ and $t_1 = 15 - M_1, \quad t_2 = 15 - M_2$
- $t_1 0$, and $t_2 0$

Therefore, we have the following

Set of feasible allocations

$\{x > 0 \quad , \quad M_1 > 0 \quad , \quad M_2 > 0\}$ (log 0 undefined $\Rightarrow$ strict inequalities)

$x + M_1 + M_2 = 30$

Pareto Optimality and Public Good

Pareto optimal allocations

A Pareto optimal allocation

A Pareto optimal allocation is a *feasible allocation* such that there is no other feasible allocation that makes at least one agent strictly better off without making someone else worse off.

Pareto Optimality and Public Good

Pareto optimal allocations

A Pareto optimal allocation

A Pareto optimal allocation is a *feasible allocation* such that there is no other feasible allocation that makes at least one agent strictly better off without making someone else worse off.

The set of the Pareto optimal allocations is the set of all feasible allocations satisfying the following

Bowen-Lindahl-Samuelson condition (B.L.S)

$$\frac{\partial U_1}{\partial x} / \frac{\partial U_1}{\partial M_1} + \frac{\partial U_2}{\partial x} / \frac{\partial U_2}{\partial M_2} = C_m(x)$$

Pareto Optimality and Public Good

Pareto optimal allocations

The set of Pareto optimal allocations (x, M_1, M_2) are such that

Pareto optimal allocations

$$\{10 < x < 20,\ M_1 = x - 10,\ M_2 = 40 - 2x\}$$

Pareto Optimality and Public Good

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Pareto optimal allocations

$$\{10 < x < 20,\ M_1 = x - 10,\ M_2 = 40 - 2x\}$$

The Pareto optimal allocation with identical contributions $(\bar{x}, \bar{M}_1, \bar{M}_2)$ must satisfy $\bar{M}_1 = \bar{M}_2$. Using the characterization of question **3**, yields

The egalitarian Pareto optimal allocation

$$(\bar{x} = \tfrac{50}{3},\ \bar{M}_1 = \tfrac{20}{3},\ \bar{M}_2 = \tfrac{20}{3})$$

Pareto Optimality and Public Good

Lindahl equilibrium

Let p_i denote the personalized price of one unit of public good for consumer i.

Lindahl equilibrium

A pair (p_1, p_2) is a Lindahl equilibrium if for $i = \{1, 2\}$ the solution x_i^* of the problem

$$\begin{cases} \max\limits_{x_i} U_i(x_i, M_i) \\ s.t \;\; p_i x_i + M_i = 15 \end{cases} \tag{4}$$

satisfies $x_i^* = x$ and $p_1 + p_2 = 1$.(constant marginal cost=1)

Pareto Optimality and Public Good

Lindahl equilibrium

Pareto Optimality and Public Good

Lindahl equilibrium

For consumer 1, the problem (4) transforms into

$$\max_{x_1} \quad 2logx_1 + log(15 - p_1x_1)$$

For consumer 2, the problem (4) transforms into

$$\max_{x_2} \quad logx_2 + 2log(15 - p_2x_2)$$

Pareto Optimality and Public Good

Lindahl equilibrium

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For consumer 2, the problem (4) transforms into

$$\max_{x_2} \quad logx_2 + 2log(15 - p_2x_2)$$

- Consumer 1's demand for the public good is thus $x_1^* = \frac{10}{p_1}$.

Pareto Optimality and Public Good

Lindahl equilibrium

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For consumer 2, the problem (4) transforms into

$$\max_{x_2} \quad logx_2 + 2log(15 - p_2x_2)$$

- Consumer 1's demand for the public good is thus $x_1^* = \frac{10}{p_1}$.
- Consumer 2's demand for the public good is thus $x_2^* = \frac{5}{p_2}$.

Pareto Optimality and Public Good

Lindahl equilibrium

Pareto Optimality and Public Good

Lindahl equilibrium

- For the pair (p_1, p_2) to be a Lindahl equilibrium, it must hold that $x_1^* = x_2^* = x \Leftrightarrow \mathrm{p}_1 = 2p_2$.

Pareto Optimality and Public Good

Lindahl equilibrium

- For the pair (p_1, p_2) to be a Lindahl equilibrium, it must hold that $x_1^* = x_2^* = x \Leftrightarrow \mathrm{p}_1 = 2p_2$.
- Besides, to satisfy the provision of the public good, it must hold that $p_1 + p_2 = C_m(x) = 1 \Rightarrow$ the price pair $(p_1 = \frac{2}{3}, p_2 = \frac{1}{3})$ is a Lindahl equilibrium

Pareto Optimality and Public Good

Lindahl equilibrium

- For the pair (p_1, p_2) to be a Lindahl equilibrium, it must hold that $x_1^* = x_2^* = x \Leftrightarrow \mathrm{p}_1 = 2p_2$.
- Besides, to satisfy the provision of the public good, it must hold that $p_1 + p_2 = C_m(x) = 1 \Rightarrow$ the price pair $(p_1 = \frac{2}{3}, p_2 = \frac{1}{3})$ is a Lindahl equilibrium

The Lindahl equilibrium allocation

The allocation $(x = 15, M_1 = 5, M_2 = 10)$ belongs to the set of the Pareto optimal allocations defined above

Pareto Optimality and Public Good

Subscription equilibrium

When each consumer decides privately on its own contribution to the provision of the public good, for $i = \{1, 2\}$ taking t_j as given consumer i solves the following problem

$$\begin{cases} \max\limits_{t_i} U_i(t_i + t_j, M_i) \\ \quad s.t \;\; t_i + M_i = 15 \end{cases} \tag{5}$$

Pareto Optimality and Public Good

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For consumer 1 the problem (5) is readily rewritten as

$$\begin{cases} \max\limits_{t_1} \quad 2log(t_1 + t_2) + log(15 - t_1) \\ \qquad\qquad\qquad\qquad s.t \;\; 0 \leq t_1 < 15 \end{cases}$$

Pareto Optimality and Public Good

Subscription equilibrium

The first-order condition yields the following

$$\frac{30 - 3t_1 - t_2}{(t_1 + t_2)(15 - t)} = 0 \Leftrightarrow t_1(t_2) = \frac{30 - t_2}{3}$$

Hence, $t_1 = \frac{30-t_2}{3}$ if $t_2 < 30$ and $t_1 = 0$ otherwise.

Pareto Optimality and Public Good

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Hence, $t_1 = \frac{30-t_2}{3}$ if $t_2 < 30$ and $t_1 = 0$ otherwise.

Similarly, for consumer 2 the F.O.C yields the following

$$\frac{15 - 2t_1 - 3t_2}{(t_1 + t_2)(15 - t)} = 0 \Leftrightarrow t_2(t_1) = \frac{15 - 2t_1}{3}$$

Hence, $t_2 = \frac{15-t_1}{3}$ if $t_1 < \frac{15}{2}$ and $t_2 = 0$ otherwise.

Pareto Optimality and Public Good

Subscription equilibrium

The resulting subscription are thus $(t_1 = 10, t_2 = 0)$. This yields the following

The (Nash) subscription equilibrium

The resulting allocation $(x = 10, M_1 = 5, M_2 = 15)$ does not belong to the set of Pareto optimal allocations.

A Market for Emission Permits

Social Optimum

Given the welfare function:

$$W(k_1, k_2) = B(n(k_1 + k_2)) - n(c_1 k_1^2 + c_2 k_2^2)$$

A Market for Emission Permits

Social Optimum

Given the welfare function:

$$W(k_1, k_2) = B(n(k_1 + k_2)) - n(c_1 k_1^2 + c_2 k_2^2)$$

the first-order conditions for welfare-maximization w.r.t k_1 and $k_2$are given by

$$\begin{pmatrix} n + c_1 & n \\ n & n + c_2 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} \hat{Q} \\ \hat{Q} \end{pmatrix}$$

A Market for Emission Permits

Social Optimum

This yields the following

Socially optimal abatement levels (with $c_2 > c_1$)

$$k_1^* = \frac{c_2 \hat{Q}}{n(c_1 + c_2) + c_1 c_2} > k_2^* = \frac{c_1 \hat{Q}}{n(c_1 + c_2) + c_1 c_2} \tag{6}$$

A Market for Emission Permits

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This yields the following

Socially optimal abatement levels (with $c_2 > c_1$)

$$k_1^* = \frac{c_2\hat{Q}}{n(c_1 + c_2) + c_1c_2} > k_2^* = \frac{c_1\hat{Q}}{n(c_1 + c_2) + c_1c_2} \tag{6}$$

Since there are n firms of each type, we have the following

Socially optimal aggregate abatement level

$$K^* = \frac{n(c_1 + c_2)\hat{Q}}{n(c_1 + c_2) + c_1c_2} \tag{7}$$

A Market for Emission Permits

Uniform Abatement Standards

The socially optimal total abatement costs (TAC) are given by:

$$TAC(k_1^*, k_2^*) = n[c_1 {k_1^*}^2 + c_2^* {k_2^*}^2] \tag{8}$$

whereas the total abatement costs (TAC) under uniform standards are given by:

$$TAC(\bar{k}, \bar{k}) = n(c_1 + c_1)\bar{k}^2 \tag{9}$$

A Market for Emission Permits

Uniform Abatement Standards

The welfare losses associated with the uniform abatement standards equal the difference in abatement costs (8)-(9).

$$\Delta W = n[c_1 {k_1^*}^2 + c_2 {k_2^*}^2] - n(c_1 + c_1)\bar{k}^2$$

A Market for Emission Permits

Uniform Abatement Standards

The welfare losses associated with the uniform abatement standards equal the difference in abatement costs (8)-(9).

$$\Delta W = n[c_1 k_1^{*2} + c_2 k_2^{*2}] - n(c_1 + c_1)\bar{k}^2$$

Using equations the expression derived in (6) and (7) yields the following

Welfare losses associated with the uniform abatement standards

$$\Delta W = -\frac{n\hat{Q}^2(c_1 + c_2)(c_1 - c_2)^2}{4[c_1 c_2 + n(c_1 + c_2)]^2}$$

A Market for Emission Permits

The demand for permits

A firm of type i's demand function for permits obtains by solving

$$\begin{cases} \min\limits_{q_i} \{\sigma q_i + c_i(\hat{q} - q_i)^2\} \\ \qquad\qquad s.t \;\; 0 \leq q_i \leq \hat{q} \end{cases}$$

A Market for Emission Permits

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The (necessary and sufficient) F.O.C yields

A firm of type i's demand function for permits

$$q^d(\sigma) = \hat{q} - \frac{\sigma}{2c_i}$$

A Market for Emission Permits

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The (necessary and sufficient) F.O.C yields

A firm of type i's demand function for permits

$$q^d(\sigma) = \hat{q} - \frac{\sigma}{2c_i}$$

Summing over the 2n firms yields

The industry aggregate demand for permits

$$Q^d(\sigma) = \hat{Q} - \frac{n(c_2 + c_1)\sigma}{2c_1c_2} \tag{10}$$

A Market for Emission Permits

The supply of permits

The regulator's supply function of permits obtains by solving

$$\begin{cases} \max_{Q}\{\sigma Q - (\hat{Q} - Q) + 2\hat{Q}(\hat{Q} - Q)\} \\ \qquad\qquad\qquad\qquad s.t \;\; 0 \leq Q \leq \hat{Q} \end{cases}$$

A Market for Emission Permits

The supply of permits

The regulator's supply function of permits obtains by solving

$$\begin{cases} \max_{Q}\{\sigma Q - (\hat{Q} - Q) + 2\hat{Q}(\hat{Q} - Q)\} \\ \qquad\qquad\qquad\qquad s.t \;\; 0 \leq Q \leq \hat{Q} \end{cases}$$

The (necessary and sufficient) F.O.C yields

The regulator's supply function of permits

$$Q^s(\sigma) = \frac{\sigma}{2} \tag{11}$$

A Market for Emission Permits

Equilibrium price and abatement levels

The equilibrium permit price is the price σ^* solution to the following market clearing condition

$$Q^s(\sigma) = Q^d(\sigma) \tag{12}$$

A Market for Emission Permits

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Solving (12) using equations(10) and (11) yields

The equilibrium permit price

$$\sigma^* = \frac{2(c_1 + c_2)\hat{Q}}{n(c_1 + c_2) + c_1 c_2}$$

A Market for Emission Permits

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$$Q^s(\sigma) = Q^d(\sigma) \tag{12}$$

Solving (12) using equations(10) and (11) yields

The equilibrium permit price

$$\sigma^* = \frac{2(c_1 + c_2)\hat{Q}}{n(c_1 + c_2) + c_1 c_2}$$

This yields the socially optimal abatement levels

A firm of type i's optimal abatement under the permits regime

$$q^d(\sigma^*) = \hat{q} - \frac{\sigma^*}{2c_i} = k_i^*$$

Thank you !