Public Economics, Welfare and Institutions

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On Commodity Subsidy

The marshllian demand and indirect utility functions

The consumer's problem is

$$\begin{cases} \max\limits_{x,y} U(x,y) = xy \\ \quad s.t \quad x + py = R \end{cases}$$

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Marshallian demand and indirect utility functions

$$Y^D(p,R) = \frac{R}{2p} \quad and \quad V(p,R) = \frac{R^2}{4p} \tag{1}$$

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The compensated (hicksian) demand

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With $R_0 = 10$ and $p_0 = 2$ this yields the following demand and indirect utility functions.

The reference outcomes

$$x_0 = \frac{10}{2}\ , \quad y_0 = \frac{10}{4}\ and\ u_0 = \frac{100}{8}$$

On Commodity Subsidy

Compensating and equivalent income variations

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Therefore, $C = 2\sqrt{p_1 u_0} - R_0$, hence using (1) yields the following:

Compensating income variation

$$C(p_0, p_1, u_0) = 2\sqrt{p_1 u_0} - 2\sqrt{p_0 u_0} < 0$$

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Compensating and Equivalent income variations

The equivalent variation E can be thought of as **the amount of money that the consumer would be indifferent about of accepting in lieu of the price change**.

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Remark: In this case, C is negative and E is positive because the price of good Y decreases thanks to the subsidy t, that is, $p_1 < p_0$.

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Compensating and Equivalent income variations

To show the results, first observe from the expression in (2) one gets

$$\int Y^C(p,u)dp = 2\sqrt{pu} \tag{3}$$

Hence,

$$\begin{aligned}\int_{p_0}^{p_1} Y^C(p,u_0)dp &= 2\sqrt{p_1u_0} - 2\sqrt{p_0u_0} \\ &= C(p_0,p_1,u_0)\end{aligned}$$

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Compensating and Equivalent income variations

Likewise, one gets

$$\int_{p_0}^{p_1} Y^C(p, u_1)dp = 2\sqrt{p_1 u_1} - 2\sqrt{p_0 u_1}$$
$$= E(p_0, p_1, u_1)$$

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Compensating and equivalent income variations

Using the expression in (1) yields the following:

$$\begin{aligned}\Delta S &= \int_{p_0}^{p_1} Y^D(p, R_0)dp \\ &= \frac{R_0}{2}\int_{p_0}^{p_1} \frac{1}{p}dp \\ &= \frac{R_0}{2}[ln(p_0) - ln(p_1)]\end{aligned}$$

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Compensating and equivalent income variations

Hence for $t = 1$ this yields the followoing:

Numerical application

- $\Delta S = 5 \log 2 \simeq 3,46$
- $C = 5(\sqrt{2} - 2) \simeq -2.33$
- $E = 10(1 - \sqrt{2}) \simeq -4.14$

This confirms numerically that $|C| < \Delta S < |E|$.

Optimality with a public good, Lindhal equilibrium

Feasible and optimal states

A state can be described as a vector with three components, (x, y_1, y_2) the ressources constraint yields

Feasible states

(x, y_1, y_2) such that $x + y_1 + y_2 = 6$

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$$xy_1 - \lambda x^2 y_2 - \mu(x + y_1 + y_2 - 6)$$

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$$xy_1 - \lambda x^2 y_2 - \mu(x + y_1 + y_2 - 6)$$

and the first-order conditions:

$$\begin{aligned} y_1 - 2\lambda x y_2 - \mu &= 0 \\ x - \mu &= 0 \\ -\lambda x^2 &= \mu \end{aligned}$$

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so that:

$$y_1 = 12 - 3x,\ y_2 = 2x - 6.$$

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Hence, one obtains the following

Pareto-optimal states

$$\{(x, y_1, y_2) : 3 \leq x \leq 4, \; y_1 = 12 - 3x, \; y_2 = 2x - 6\}$$

Optimality with a public good, Lindhal equilibrium

Lindhal equilibrium

Maximization of individual utility under the budget constraint gives:

$$\begin{aligned} x_1^d &= (4+T_1)/(2p_x^1) \\ y_1^d &= (4+T_1)/2 \end{aligned}$$

and

$$\begin{aligned} x_2^d &= 2(2+T_2)/(3p_x^2) \\ y_2^d &= (2+T_2)/3 \end{aligned}$$

Optimality with a public good, Lindhal equilibrium

Lindhal equilibrium

For this allocation to be a Lindahl equilibrium, it must holds that

- $x_1^s = x_2^s = y_0$, where y_0 is the quantity of input for the producer and x_i^s the supply.

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- $x_1^s = x_2^s = y_0$, where y_0 is the quantity of input for the producer and x_i^s the supply.
- $p_x^1 + p_x^2 = 1$ since the producer's profit is given by :$(p_x^1 + p_x^2 - 1)y_0$.

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The market equilibrium is such that demand and supply are equal. That is,
$x_1^d = (4 + T_1)/(2p_x^1) = x_1^s = y_0$, so that

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For the private good, the feasability constraint yields

$$y_0 + y_1 + y_2 = 6$$

Optimality with a public good, Lindhal equilibrium

Transfers such that for $T = T_1 = -T_2$

$$p_x^1 = \frac{3(4+T)}{20-T},\ p_x^2 = \frac{4(2-T)}{20-T}$$

and

$$y_1 = (4+T)/2,\ y_2 = (2-T)/3.$$

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Setting $T = 20 - 6x$, one obtains $y_1 = 12 - 3x$ and $y_2 = 2x - 6$, and thus the Pareto states.

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For $T = 0$, $y_1 = 2$, $y_2 = 2/3$ and $x = 10/3$ is one of these optima.

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Important remark

These results illustrate the Second Fundamental Theorem of Welfare Economics

Thank you !