Time Discounting in Intertemporal Cost-Benefit Analysis

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Introduction: climate change as a motivational issue

- The issue of climate change is prominent in the public debate: Grenelle de l'environnement, Copenhagen and Cancún meetings, etc.
- The difficulty is that we have to compare current costs with future long-run (and uncertain) costs.
- There have been many controversies in the economic literature on the severity of the problem. For instance, how much should we reduce greenhouse gases emissions?

Introduction: the standard evaluation method

- The standard method to evaluate environmental policy is cost-benefit analysis: present gains vs. future lost.
- For instance, the social cost of carbon (used to design the carbon tax): the sum of all future costs due to the emission of an extra ton of carbon.
- In the intertemporal context, the implicit price of future costs is embodied in the practice of discounting: a loss of consumption $-dc_t$ tomorrow is equivalent to a loss of $-e^{-\rho t}dc_t$ today, where ρ is the discount rate.

Introduction: illustration of the standard method

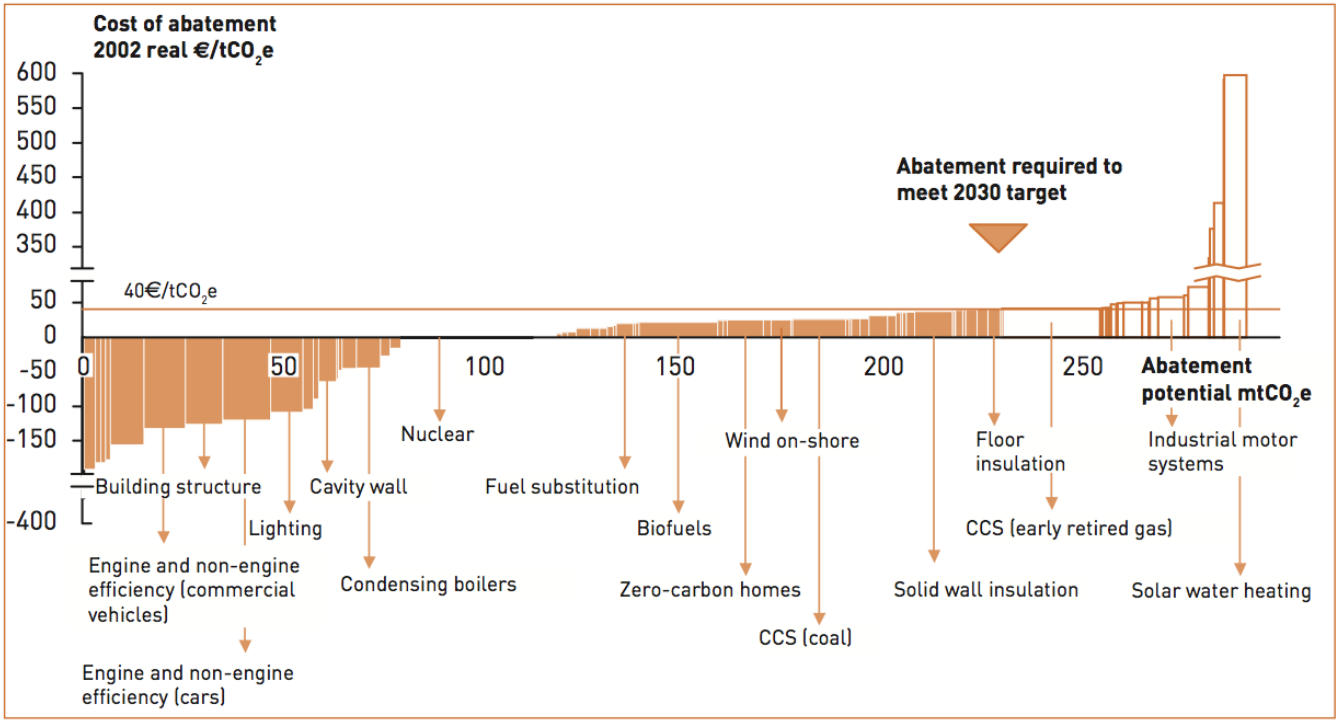

Figure: Abatement cost in th UK (source: McKinsey)

Public policy use of the social cost of carbon: the carbon tax

- The carbon tax can be conceived as a Pigouvian tax used to have agents internalizing the cost of the externality they exert. If the social cost of carbon is the cost of the climate externality, the carbon tax should be equal to the social cost of carbon.

- Recommendation of values for the social cost of carbon for public policies/the carbon tax have been made in several countries:
  - In the UK, the recommended carbon tax in 2010 was 27.30£= 30.5€ (DEFRA).
  - In France, the recommended carbon tax in 2010 was 32€ (Rapport Quinet).
  - In the US, the recommended carbon tax in 2010 was 21$=15€ (US Department of Energy).

THE STERN/NORDHAUS CONTROVERSY

- Recent debates on the economics of climate change were triggered by the Stern review (2006). Prompted strong actions to prevent climate change.

- There was a controversy with Nordhaus (JEL 2007) who on the contrary recommended a gradual response in terms of reducing emissions.

- The main difference turned out to be the chosen value of ρ in the Ramsey equation

$$\rho = \delta + \eta g$$

They interpreted the parameters (in particular δ and η) in very different ways.
Stern used $\delta = 0.1$, $\eta = 1$ and $g = 2$ while Nordhaus used $\delta = 3(1.5)$, $\eta = 1(1.5)$ and $g = 2$.

Questions

- How can research in theoretical economics contribute to the debate?
- To provide guidance for the choice of δ and η, explicit the theoretical basis for the Ramsey formula.
  $\Longrightarrow$ axiomatization of social preferences.
- Discuss other possibilities for the discount rate.

The axiomatic method

- The axiomatic approach characterizes (classes of) preferences from a set of properties (axioms).
- The aim is
  1. to delineate possibilities and/or impossibilities for a set of desirable properties.
  2. to know what principles certain classes of preferences embody (to assess their reasonableness).
- In the case of cost-benefit analysis, it is important to check that the social preferences used have nice properties so that the policy recommendations made have sound axiomatic (ethical/rational) foundations.

Plan of the talk

- Introduce the standard method to cost benefit analysis and discounting, and show that it relies on a specific criterion.
- Discuss how considerations of fairness and equity can be introduced in the analysis and what they change.
- Discuss the impact of risk, catastrophes and aversion to catastrophes.
- Conclude.

Evaluating marginal changes

- To evaluate policies, it is common to use cost-benefit analyses dealing with marginal changes.
- Suppose that the society is aiming at maximizing

$$\sum_{t=0}^{+\infty} \beta^t u(c_t)$$

and consider a small perturbation dc_t around the path c_t, with $dc_0 > 0$ and $dc_t < 0$ for $t > 0$.
- The project is worth implementing if

$$\sum_{t=0}^{+\infty} \beta^t u(c_t + dc_t) - \sum_{t=0}^{+\infty} \beta^t u(c_t) \geq 0$$

which depends on wether

$$dc_0 \geq -\sum_{t=1}^{+\infty} \beta^t \frac{u'(c_t)}{u'(c_0)} dc_t$$

The social discount rate

- Denote $\rho_t(c) = -\ln\left(\beta \frac{u'(c_{t+1})}{u'(c_t)}\right)$ and remark that

$$\beta^t \frac{u'(c_t)}{u'(c_0)} = \prod_{\tau=0}^{t-1} \beta \frac{u'(c_{\tau+1})}{u'(c_\tau)} = e^{-\sum_{\tau=0}^{t-1} \rho_\tau(c)}.$$

- The evaluation of the future cost of the policy (e.g. the cost of carbon), $\sum_{t=1}^{+\infty} \beta^t \frac{u'(c_t)}{u'(c_0)} dc_t$, can be written

$$\sum_{t=1}^{+\infty} e^{-\sum_{\tau=0}^{t-1} \rho_t(c)} dc_t.$$

- $\rho_t(c)$ is the social discount rate at period t.
  In general, depends on t and on the path c.

The social discount rate: an approximation

- Remark that

$$\begin{aligned}\ln\left(\frac{u'(c_{t+1})}{u'(c_t)}\right) &= \ln\left(\frac{u'(c_t+c_{t+1}-c_t)}{u'(c_t)}\right)\\ &\approx \ln\left(1+\frac{u''(c_t)(c_{t+1}-c_t)}{u'(c_t)}\right)\\ &\approx \frac{u''(c_t)c_t}{u'(c_t)}\cdot\frac{c_{t+1}-c_t}{c_t}\end{aligned}$$

- Denoting $\delta = -\ln(\beta)$, $\eta_t(c) = -\frac{u''(c_t)c_t}{u'(c_t)}$ and $g_t(c) = \frac{c_{t+1}-c_t}{c_t}$, we obtain the approximation

$$\rho_t(c) \approx \delta + \eta_t(c)g_t(c)$$

- How to interpret the 'parameters' in the above equations?
  - δ = pure time discounting. Problem: source?
  - $\eta_t(c) = 1/\sigma_t(c)$, where σ_t is the intertemporal elasticity of substitution. Also interpreted as 'risk aversion' and 'inequality aversion' (see the function $u(c) = c^{1-\eta}/(1-\eta)$).

A Foundation for the social discount rate: the mainstream approach

- Mainstream approach to social discounting rely on the assumption of a representative agent having preferences similar to "well-behaved"individual preferences.
- This justifies the use of market rates.
  Problem though:
  - which market rates?
  - Are market working well in that case?
- Standard preferences can be traced back to Samuelson (1937).

A Foundation for the social discount rate: Koopmans (1)

- The seminal contribution is Koopmans (1960).
- Koopmans' axiomatization relies on three main principles:
  - the evaluation is independent of the past, independent of the future.
  - the evaluation is stationary:

$$(c_0, c_1, c_2, \ldots) \succeq (c_0, c_1', c_2', \ldots) \iff (c_1, c_2, \ldots) \succeq (c_1', c_2', \ldots)$$

  - (the evaluation is time consistent and/or time invariant).
- These principles imply a recursive evaluation function:

$$U(c_0, c_1, c_2, \ldots) = \phi\Big(u(c_0), U(c_1, c_2, \ldots)\Big)$$

A Foundation for the social discount rate: Koopmans (2)

- To obtain the usual additive separable formula Koopmans added an axiom of period independence:

$$(c_0, c_1, c_2, \ldots) \succeq (c_0', c_1', c_2, \ldots) \iff (c_0, c_1, c_2', \ldots) \succeq (c_0', c_1', c_2', \ldots)$$

- In that case it can be showed that the aggregator function ϕ must be linear:

$$U(c_0, c_1, c_2, \ldots) = u(c_0) + \beta U(c_1, c_2, \ldots)$$

- By time invariance, we obtain the standard additive model with exponential discounting:

$$U(c_0, c_1, c_2, \ldots) = \sum_{t=0}^{+\infty} \beta^t u(c_t)$$

Recursive vNM utilities: Epstein (1)

- Epstein (1983) extended the axiomatization by Koopmans to situations involving risk and decision makers having von Neumann Morgenstern (vNM) preferences.
- He used the same principles as Koopmans, especially the stationarity axiom (restated for the case of lotteries):

$$(c, l) \succeq (c, l') \iff l \succeq l'$$

where l and l' are lotteries over infinite utility streams $(c_0, c_1, c_2, \ldots$

Recursive vNM utilities: Epstein (2)

In the risky case, Koopmans' principles imply a recursive vNM evaluation function more specific than in the deterministic case

$$U(c_0, c_1, c_2, \ldots) = u(c_0) + \beta(c_0)U(c_1, c_2, \ldots)$$

so that

$$U(c_0, c_1, c_2, \ldots) = \sum_{t=0}^{+\infty} u(c_t) \exp\left(-\sum_{\tau=0}^{t-1} v(c_\tau)\right)$$

where $v(c_t) = -\ln(\beta(c_t))$.

Proof.

Recursive vNM utilities: Epstein (3)

When the function v is constant, we are back to the standard additive separable case with exponential discounting.

This is the case as soon as one further assumes the period independence axiom of Koopmans.

What to remember from the mainstream approach

- The main idea is that of a representative agent having 'rational' preferences with nice properties (stationarity, invariability, time separability).

- Problems:
  - No explicit foundation of the representative agent: it is very dubious he can have stationary preferences.
  - The empirical validity of the proposed properties is (at least) questionable.

- Interpretations of the parameters δ and η in the SDR formula ($\rho = \delta + \eta g$): the rate of pure time preference and the inverse of the intertemporal elasticity of substitution.
  The best is to use an appropriate market rate (Nordhaus).

Equity and fairness: introduction

- Arguments of fairness are very often used in the discussions surrounding climate change.
- There are different aspects of the equity issue:
  - Equity between generations : impartiality and distributive equity.
  - Equity within generations : equity weights.

Intergenerational fairness: pure discount rate

- The assumption that $(0 <)\beta < 1$ has been criticized by economists and philosophers for many years (Sidgwick, Ramsey, Rawls,...):
  In the intergenerational context, it means that less weight is put on the utility of future generations, a violation of impartiality.

- But one explanation for utility discounting has been proposed by the same authors (+ Dasgupta and Heal; and Stern): the risk that future generations may not exist (see below).

- Apart from this case, there is no *normative* argument to use $\beta > 0$.

Discounting as a consequence of inequality aversion

- In the standard formula the discount factor β^t puts less weight on the utility of future generations.
  But if future generations are rich, this can be interpreted as welfare weights putting more weight on the utility of the poor (like Gini weights).

- On can propose the following social welfare function:

$$\sum_{t=0}^{\infty} \beta^t u(c_{[t]})$$

where $c_{[t]}$ is a reordering of $(c_0, c_1, \ldots)$ in increasing sequence.

Discounting as a consequence of inequality aversion (continued)

- Features of the social welfare function $\sum_{t=0}^{\infty} \beta^t u(c_{[t]})$:
  - The social welfare function satisfies Pareto (on a subdomain) and impartiality.
  - The concavity of u and β jointly determines inequality aversion.

- If future generation are rich, a more inequality averse society discounts more future costs (lower social cost of carbon).
  But if some future generation is less well-off than the current one (because of climate change), strong action should be undertaken to limit their damages.

Welfare weights

- Assume there are N country and that the social welfare function is:

$$\sum_{t=0}^{+\infty} \beta^t \sum_{i=1}^{N} n_{it} \frac{c_{it}^{1-\eta}}{1-\eta}$$

If costs and benefits are the same for all countries, the social cost of carbon is:

$$SCC = \sum_{t=0}^{+\infty} \beta^t \bar{c}_t^{-\eta} \sum_{i=1}^{N} n_{it} \left(\frac{c_{it}}{\bar{c}_t}\right)^{-\eta} dc_t$$

- On can redefine the social discount rate as

$$\rho_t = -\ln\left(\frac{\beta \bar{c}_{t+1}^{-\eta} \sum_{i=1}^{N} n_{it+1} \left(\frac{c_{it+1}}{\bar{c}_{t+1}}\right)^{-\eta}}{\bar{c}_t^{-\eta} \sum_{i=1}^{N} n_{it} \left(\frac{c_{it}}{\bar{c}_t}\right)^{-\eta}}\right)$$

Welfare weights: continued

- Hence

$$\rho_t = -\ln(\beta) + \eta \ln\left(\frac{\bar{c}_{t+1}}{\bar{c}_t}\right) + \eta \ln\left(\frac{\left(\sum_{i=1}^{N} n_{it+1}\left(\frac{c_{it+1}}{\bar{c}_{t+1}}\right)^{-\eta}\right)^{-1/\eta}}{\left(\sum_{i=1}^{N} n_{it}\left(\frac{c_{it}}{\bar{c}_t}\right)^{-\eta}\right)^{-1/\eta}}\right)$$

- $E_{1+\eta,t} = \left(\sum_{i=1}^{N} n_{it}\left(\frac{c_{it}}{\bar{c}_t}\right)^{-\eta}\right)^{-1/\eta}$ is an equality index. Denoting g_t^E the growth rate of equality, we obtain:

$$\rho_t \approx \delta + \eta g_t + \eta g_t^E$$

- When inequality decreases, discounting increases and hence the social cost of carbon decreases.

Welfare weights: continued 2

- Now the costs and benefits may not be shared equally among the different countries. Let α_{it} be the relative cost (or benefit) in country i and period t with respect to average cost (or benefit). The social cost of carbon is:

$$SCC = \sum_{t=0}^{+\infty} \beta^t \bar{c}_t^{-\eta} \sum_{i=1}^{N} n_i \alpha_{it} \left(\frac{c_{it}}{\bar{c}_t} \right)^{-\eta} d\bar{c}_t$$

- On can redefine the social discount rate as

$$\rho_t = -\ln \left(\frac{\beta \bar{c}_{t+1}^{-\eta} \sum_{i=1}^{N} n_{it+1} \alpha_{it+1} \left(\frac{c_{it+1}}{\bar{c}_{t+1}} \right)^{-\eta}}{\bar{c}_t^{-\eta} \sum_{i=1}^{N} n_{it} \alpha_{it} \left(\frac{c_{it}}{\bar{c}_t} \right)^{-\eta}} \right)$$

Welfare weights: continued 3

- The social discount rate (and the SCC) critically depend on the α_{it}.
- However, we have the following general result (Fleurbaey and Zuber, 2012) in the very long run ($t \to \infty$):

$$1 + \rho_t \approx \max_i \min_j \left(1 + \rho_t^{i,j}\right)$$

where $\rho_t^{i,j}$ is the social discount rate that would prevail if all costs were supported by country i and all benefits received by country j.

Equity weights and SCC (in 2015)

	$\eta = 0$	$\eta = 1$	$\eta = 1.5$	$\eta = 2$
$\beta = 0.00$	8752	937	257	76
$\beta = 0.01$	1305	206	75	30
$\beta = 0.015$	619	119	48	22
$\beta = 0.03$	126	37	19	10

Reduction costs paid by all countries

	$\eta = 0$	$\eta = 1$	$\eta = 1.5$	$\eta = 2$
$\beta = 0.00$	8752	3078	2234	2061
$\beta = 0.01$	1305	677	650	818
$\beta = 0.015$	619	391	421	588
$\beta = 0.03$	126	122	163	274

Reduction costs paid by rich countries

Note: SCC in 2015 in $ taken from Nordhaus (2012).

The social discount rate: introducing risk

- The future evolution of the climate (and the whole economy) is plagued with deep uncertainties.
- These uncertainties affect the valuation of future costs, hence the social discount rate and the SCC.
- Two kind of uncertainties have been put forward:
  - The uncertainty about the future level of consumption
  - The uncertainty about the length of the planning horizon

Risky consumption path: standard approach

- Simple two-periods model, the social welfare function is:

$$W(c) = u(c_0) + \beta E u(\tilde{c}_1)$$

with $u(c) = c^{1-\eta}/(1-\eta)$. The variable $\tilde{X} = \ln \tilde{c}_1 - \ln c_0$ is assumed to be normally distributed.

- The discount rate ρ can be defined as before:

$$\rho = -\ln\left(\frac{\beta E \tilde{c}_1^{-\eta}}{c_0^{-\eta}}\right) = \delta - \ln\left(E e^{-\eta \tilde{X}}\right)$$

- As shown by Gollier (2008), $E e^{-\eta \tilde{X}} = \exp\left(-\eta\left(E\tilde{X} - 0.5\eta Var(\tilde{X})\right)\right)$ so that:

$$\rho = \delta + \eta g - 0.5\eta(\eta+1)Var(\tilde{X})$$

where $g = \ln(E\tilde{c}_1/c_0) = E\tilde{X} + 0.5Var(\tilde{X})$

Risky consumption path: Weitzman's (2009) dismal theorem

- Weitzman (2009) uses the same model as Gollier, except that the variable $\tilde{X} = \ln \tilde{c}_1 - \ln c_0$ is normally distributed *but with an uncertain variance*.

- Using bayesian updating to infer the law of variable $\tilde{X}$, he shows that the growth rate has a fat-tail distribution (Student-t law). Weitzman indicates that the social discount rate formula for standard iso-elastic discounted utilitarian criteria implies

$$\rho = -\infty$$

- Weitzman's result has however been criticized because it critically hinges on consumption being as close to zero as one wishes and marginal utility being infinite in zero.

The risk of the world ending: setting

- Define $X = \mathbb{R}_+ \cup \{d\}$, where d denotes the non-existence status. This is the set of outcomes for generations: if the generations is alive, it consume $c \in \mathbb{R}_+$ otherwise it does not exist.

- Consider $S = \Big\{(x_0, x_1, \ldots) \in X^{\mathbb{N}} \setminus \mathbb{R}_+^{\mathbb{N}} : x_t = d \Rightarrow \forall s > 0,\ x_{t+s} = d\Big\}$.
  We will consider social preferences over $L(S)$, the set of lotteries over S.

- S is a stationary set:

$$(x_0, x_1, x_2, \ldots) \in S \Longrightarrow (x_1, x_2, \ldots) \in S$$

Bommier's (2006) extension of Epstein's result

- When social preferences over $L(S)$ satisfy the vNM axioms and stationarity, the vNM utility function is uniquely defined (up to an affine transformation) as

$$U(x_0, x_1, x_2, \ldots) = \sum_{t=0}^{+\infty} u(x_t) \exp\left(-\sum_{\tau=0}^{t-1} v(x_\tau)\right)$$

- Difference with Epstein: Epstein result excluded the case $u(c)/(1 - e^{v(c)})$ constant (to ensure convergence) ; here the condition is not required.

Multiplicative preferences

- If one adds the Anonymity principle (same treatment for all generations when they exist), one obtains *multiplicative preferences*:

$$U(x_0, x_1, x_2, \ldots, x_T) = \frac{1+\exp\left(k\sum_{t=0}^{+\infty} w(x_t)\right)}{k} = \frac{1+\prod_{t=0}^{+\infty}\left(1+ku(x_t)\right)}{k}$$

  where $w = v/k$.

- Additive preferences can be viewed as the limit case of multiplicative preferences when $k \to 0$.

- Two cases are possible:
  - $k < 0$: catastrophe avoidance.
  - $k > 0$: risk equity.

An application to an uncertain planning horizon (Bommier Zuber, 2008)

- Each period, there is a probability p that the world ends. If the world survives, consumption is certain.
- The expected utility for the multiplicative preferences is:

$$
\begin{aligned}
EU(x_0, x_1, x_2, \ldots) &= \sum_{t=0}^{+\infty} p(1-p)^t \frac{1+\prod_{\tau=0}^{t}\left(1+ku(c_\tau)\right)}{k} \\
&= (1-p)\sum_{t=0}^{+\infty} u(c_t) \prod_{\tau=0}^{t-1}\left[(1-p)\left(1+ku(c_\tau)\right)\right]
\end{aligned}
$$

The social discount rate

- Like in the standard case, the social discount rate is defined as

$$\rho_t(c) = -\ln\left(\frac{\partial EU/\partial c_{t+1}}{\partial EU/\partial c_t}\right)$$

- Denoting $D_t = \prod_{\tau=0}^{t-1}\Big[(1-p)\big(1+ku(c_\tau)\big)\Big]$, we obtain:

$$\begin{aligned}
\rho_t(c) &= -\ln\left(\frac{u'(x_{t+1})D_{t+1}+\sum_{T=t+2}^{+\infty}u(c_T)D_T\frac{ku'(c_{t+1})}{1+ku(c_{t+1})}}{u'(c_t)D_t+\sum_{T=t+1}^{+\infty}u(c_T)D_T\frac{ku'(c_t)}{1+ku(c_t)})}\right)\\
&= -\ln\left(\frac{D_{t+1}}{D_t}\cdot\frac{u'(c_{t+1})}{u'(c_t)}\cdot\underbrace{\frac{1+\frac{k}{1+ku(c_{t+1})}\sum_{T=t+2}^{+\infty}u(c_T)D_T/D_{t+1}}{1+\frac{k}{1+ku(c_t)}\sum_{T=t+1}^{+\infty}u(c_T)D_T/D_t}}_{\chi}\right)
\end{aligned}$$

The social discount rate (continued)

- Hence:

$$\begin{aligned}\rho_t(x) &= -\ln\Big((1-p)\big(1+ku(c_t)\big)\Big) - \ln\left(\frac{u'(c_{t+1})}{u'(c_t)}\right) - \ln(\chi) \\ &\approx p - ku(c_t) + \eta_t(c)g_t(c) - \ln(\chi)\end{aligned}$$

- Neglecting the term $\ln(\chi)$ (which is almost null when utility is not changing too fast), we obtain a foundation for the term $\delta \approx p - ku(c_t)$ which is not based on pure time discounting / intergenerational inequity.

- Some consequences:
  - Standard utilitarianism should discount at rate $\rho_t(c) \approx p + \eta_t(c)g_t(c)$ (Stern's proposal).
  - The higher the preference for catastrophe avoidance, the higher the discount rate.

Introducing inequality aversion under uncertainty

- Standard result in social choice under uncertainty: Harsanyi's theorem.
  - VNM preferences + Pareto principle $\Longrightarrow$ linear aggregation.
  - Does not allow to express priority for the worst-off (Diamond, 1967).
- To escape this drawback, Fleurbaey (2010) proposes to weaken Pareto's principle: Pareto for equal risk ; Pareto for no risk.
- He suggests to use the expected value of the equally distributed equivalent (EEDE).

The Expected equally distributed equivalent (EEDE)

- EDE:

$$U(c_0, c_1, c_2, \ldots, c_T) = \phi^{-1}\left(\frac{1}{T}\sum_{t=0}^{T}\phi \circ u(c_t)\right)$$

- We can make the same assumptions as above (each period, a probability p that the world ends). The EEDE is:

$$EU(c_0, c_1, c_2, \ldots) = \sum_{t=0}^{+\infty} p(1-p)^t \phi^{-1}\left(\frac{1}{T}\sum_{\tau=0}^{t}\phi \circ u(c_\tau)\right)$$

The social discount rate for the EEDE family

Using the same method as before, in the case of the EEDE family, we obtain:

$$\rho_t(x) \approx p + \eta_t(c) g_t(c) - \ln\left(\frac{\phi'(u(c_{t+1}))}{\phi'(u(c_t))}\right) - \ln\left(\frac{E_{t+1}\nu_T}{E_t\nu_T}\right)$$

where $\nu_T = [T\phi'(EDE_T)]^{-1}$.

A welfarist restatement: Impartiality and Pareto axioms

- The Anonymity is an (ethically) appealing principle that rules out pure time discounting.
- The Pareto axiom is key to intergenerational ethics
  - The standard Pareto principle

$$\left\{Eu(x_t) \geq Eu(y_t)\right\} \forall t \Longrightarrow EU(x_0, x_1, x_2, \cdots) \geq EU(y_0, y_1, y_2, \cdots)$$

  - Standard preferences (with no pure time discounting) are characterized by three principles: Pareto, Anonymity and independence of the existence of past generations.
  - No preference for utility redistribution or for the (non-)correlation of generations' fates.
- Pareto, catastrophe avoidance and redistribution:
  - Multiplicative preferences: Pareto for individual risks.
  - EEDE: Pareto for equal risks, ex post Pareto and separability.

Conclusion: the social cost of carbon

- The computation of the social cost of carbon heavily relies on the underlying social evaluation criterion and the choice of the parameters.
- To decide on the parameters, we clearly need to choose the normative basis for the social discount rate. There are still confusions in the literature.
- The additive model is very restrictive. Non additive models provide new reasons (not) to discount. For the moment, no estimates on how it could change the social cost of carbon.
- **The bottom-line: the social cost is not an 'ethic-free' number**.

Conclusion: limitations of the discount rate approach

- The use of the discounted sum of damages is correct for marginal policies only. For large-scale policies, we need to rely directly on social criteria.
- The social discount rate depends on the distribution of costs and benefits: not one single discount rate that could be used for all public policies.
- The methodology applies only for certain consequences of policies, viz changes in income/consumption. Alternative methodologies must be developed to assess changes in risks and changes in the size of the population.

Sketch of the proof of Epstein's result

- The proof hinges on the fact that vNM utility functions are unique up to a positive linear transformation.
- By stationarity, $U(c_1, c_2, c_3, \ldots)$ and $U(c_2, c_3, \ldots)$ are two vNM utility functions over infinite streams. So there exists two numbers $\beta_{c_1} > 0$ and u_{c_1} such that

$$U(c_1, c_2, c_3, \ldots) = u_{c_1} + \beta_{c_1} U(c_2, c_3, \ldots)$$

This is true for any c_1, so we can write

$$U(c_1, c_2, c_3, \ldots) = u(c_1) + \beta(c_1) U(c_2, c_3, \ldots)$$

Back.